

YEAR 10 — USING NUMBER...

Indices & Roots

What do I need to be able to do?

By the end of this unit you should be able to:

- Identify square and cube numbers
- Calculate higher powers and roots
- Understand powers of 10 and standard form
- Know the addition and subtraction rule for indices
- Understand power zero and negative indices
- Calculate with numbers in standard form

Keywords

Standard (index) Form: A system of writing very big or very small numbers

Commutative: an operation is commutative if changing the order does not change the result

Base: The number that gets multiplied by a power

Power: The exponent — or the number that tells you how many times to use the number in multiplication

Exponent: The power — or the number that tells you how many times to use the number in multiplication

Indices: The power or the exponent

Negative: A value below zero.

Coefficient: The number used to multiply a variable

Square and cube numbers

Square numbers

1, 4, 9, 16...

$$144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3$$

$$(2 \times 2 \times 3) \times (2 \times 2 \times 3)$$

12 x 12

Prime factors can find square roots

$$\sqrt{144} = 12$$

Cube numbers

1, 8, 27, 64, 125...

$$216 = 2 \times 2 \times 2 \times 3 \times 3 \times 3$$

$$(2 \times 3) \times (2 \times 3) \times (2 \times 3)$$

6 x 6 x 6

$$\sqrt[3]{216} = 6$$

Higher powers and roots

$$x^n$$

x — the base number

n — power
(number of times multiplied by itself)

$$\sqrt[n]{x}$$

Finding the nth root of any value

Other mental strategies for square roots

$$\begin{aligned}\sqrt{810000} &= \sqrt{81} \times \sqrt{10000} \\ &= 9 \times 100 \\ &= 900\end{aligned}$$

Standard form

Any number between 1 and less than 10

$$A \times 10^n$$

Any integer

Example

$$\begin{aligned}3.2 \times 10^4 \\ &= 3.2 \times 10 \times 10 \times 10 \times 10 \\ &= 32000\end{aligned}$$

Non-example

$$\begin{aligned}(0.8) \times 10^4 \\ 5.3 \times 10^{07}\end{aligned}$$

Any value to the power 0 always = 1

Numbers in standard form with negative powers will be less than 1

$$3.2 \times 10^{-4} = 3.2 \times \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} = 0.00032$$

Negative powers do not indicate negative solutions

Zero and negative indices

$$x^0 = 1$$

Any number divided by itself = 1

$$\frac{a^6}{a^6} = a^6 \div a^6$$

$$= a^{6-6} = a^0 = 1$$

Negative indices do not indicate negative solutions

$$\begin{aligned}2^2 &= 4 \\ 2^1 &= 2 \\ 2^0 &= 1\end{aligned}$$

$$2^{-1} = \frac{1}{2}$$

$$2^{-2} = \frac{1}{4}$$

Looking at the sequence can help to understand negative powers

Powers of powers

$$(x^a)^b = x^{ab}$$

$$(2^3)^4 = 2^3 \times 2^3 \times 2^3 \times 2^3$$

The same base and power is repeated Use the addition law for indices

$$(2^3)^4 = 2^{12} \leftarrow a \times b = 3 \times 4 = 12$$

NOTICE the difference

$$(2x^3)^4 = 2x^3 \times 2x^3 \times 2x^3 \times 2x^3$$

The addition law applies ONLY to the powers. The integers still need to be multiplied

$$(2x^3)^4 = 16x^{12}$$

Standard form calculations

Addition and Subtraction

Tip: Convert into ordinary numbers first and back to standard form at the end

Method 1

$$\begin{aligned}6 \times 10^5 + 8 \times 10^5 \\ &= 600000 + 800000 \\ &= 1400000 \\ &= 1.4 \times 10^6\end{aligned}$$

Multiplication and division

$$\begin{aligned}\frac{1.5 \times 10^5}{0.3 \times 10^3} &\leftarrow \text{Division questions can look like this} \\ &= (1.5 \times 10^5) \div (0.3 \times 10^3) \\ &= (15 \div 0.3) \times 10^{5-3} \\ &= 5 \times 10^2\end{aligned}$$

Method 2

$$\begin{aligned}&= (6 + 8) \times 10^5 \\ &= 14 \times 10^5 \\ &= 1.4 \times 10^1 \times 10^5 \\ &= 1.4 \times 10^6\end{aligned}$$

This is not the final answer

For multiplication and division you can look at the values for A and the powers of 10 as two separate calculations

Addition/ Subtraction Laws

$$a^m \times a^n = a^{m+n}$$

$$a^m \div a^n = a^{m-n}$$